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RESEARCH ON TIME-VARYING RELIABILITY OF WIND POWER GEARBOX WITH FAILURE CORRELATION

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In the process of reliability modeling and design of a wind power gearbox, the influence of failure correlation of system components and random dynamic loads can not be ignored. In this paper, taking the output power of each shaft of the gearbox as the starting point, distribution parameters of stress and strength for critical failed components (gear and bearing) of the gearbox are solved. After normalization of these parameters, and when there is a linear correlation between the loads of each component, finally, the time-varying reliability related to failure correlation is calculated based on a full probability differential method by using the stress strength interference theory at the system level, and the results are verified by the Monte-Carlo simulation method. It is found that the failure correlation caused by the external dynamic loads between components can significantly reduce reliability of the gearbox system. The stronger the correlation, the lower the reliability of the gearbox system. Meanwhile, the time-varying reliability conforms to characteristics of the "failure bathtub curve" of typical mechanical products. This paper can lay a foundation for the time-varying reliability optimization design and structural robustness design of the wind power gearbox.

Keywords: wind power gearbox, failure correlation, stress-strength, time-varying reliability

1. Introduction

Wind power is a renewable energy with great resource potential and mature technology. The gearbox in the wind power transmission system is a necessary and high cost component in the whole wind turbine. Its main function is to realize transmission of energy from the impeller to the generator and change the transmission speed. Its reliability has always been a major problem in the wind power industry. Qin *et al.* (2012) deeply studied gearbox reliability considering the coupled action of gear and bearing under random dynamic loads. They used the test coefficient distribution method to deal with failure correlation, and reliability calculation depended on dynamical equations. Lewis (2001) proposed an approximate numerical method for estimating the static failure probability of a gearbox by using the stress-strength interference model. Considering the time-varying effect of loads, Wang *et al.* (2014) studied the dynamic reliability of a gear transmission system of a wind turbine. Giger and Armando (2011) revised the structure of a wind power gearbox for improving reliability, but this structure was difficult to manufacture and its life could not be calculated quantitatively. Zhou *et al.* (2013) only evaluated the dynamic reliability of a wind power gearbox according to correlation between two failure modes of a pair of gears. They only aimed at meshing of a pair of gears and did not study reliability of the system. Ditlevsen (1979) proposed a second-order reliability interval estimation method by considering the correlation between two failure modes of the whole gearbox, but calculation of the correlation coefficient was empirical. The latest research on the reliability of a wind power gearbox was made by Guo *et al.* (2020), Bhardwaj *et al.* (2019), Jiang *et al.* (2018), Liu *et al.* (2016), Bejger *et al.* (2021), Hosseinzadeh and Salmasi (2016), Röder *et al.* (2021) and so on. Although the above references adopted reliability analysis and structural optimization design of a gearbox from different aspects, and achieved some corresponding theoretical results, but there are few studies considering component failure correlation and system time-varying reliability at the same time.

Due to complexity of the internal and external excitation and the correlation between the coupling effects of various components in the wind power gearbox system, stress of the gear and bearing is time-varying and has failure correlation. The data in the references (Jantara *et al.*, 2020; Mo *et al.*, 2018) show that the reliability of the gearbox is dynamic and has attenuation characteristics, and the data in the references (Inturi *et al.*, 2022; Srinivasan and Robert, 2021) show that the parts of the whole gearbox system are not independent, and there is a certain correlation between failure of gearbox parts. So the time-varying reliability of a wind power gearbox with failure correlation is more accurate and in line with the actual situation. In this paper, taking a MW type of a wind power gearbox as the research object, we establish a time-varying reliability evaluation model considering failure correlation. Specifically, using relevant standards of the mechanical design, the distribution of parameters determines the dynamic meshing force of each gear pair and the dynamic contact force of each support bearing. On this basis, according to the stress-strength interference model at the system level, and using a sequential statistical method in a random process, the law of change with time of the gearbox reliability considering multiple actions of a random load and failure correlation is solved. The accurate evaluation of the reliability of the wind power gearbox system considering failure correlation has great practical significance for safe operation and maintenance strategy of the whole wind turbine.

2. Model of the wind power gearbox

Because the planetary gear transmission adopts multiple planetary gears to share the load and realize power diversion, it has an advantage of small volume, light weight and strong bearing capacity. However, considering the relatively complex structure and difficult processing of a large internal gear ring, the transmission structure with one-stage planetary plus two-stage parallel shafts (NGW type) is usually adopted for a large wind power gearbox. Its transmission structure (Chen *et al.*, 2011) is shown in Fig. 1.

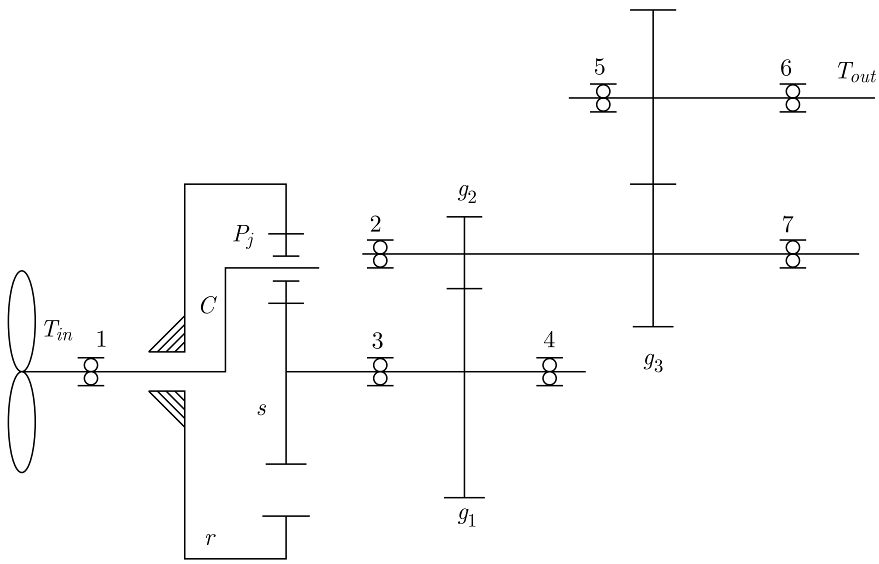

Fig. 1. A sketch of a wind power gear transmission system

Here, the planet carrier is the input and the inner gear ring is fixed, r, s, p, c, g, T_{in}, T_{out} represent the inner gear ring, solar gear, planetary gear, planetary frame, helical gear, input torque and output torque, respectively.

According to the main failure modes of gears and rolling bearings in the wind power gearbox, gears usually adopt a hard tooth surface, so wear and plastic deformation of the gear tooth surface are not the main causes of gearbox failure. In this paper, the reliability of the gear tooth root bending fatigue strength is evaluated. The reliability of the rolling bearing is calculated according to the pitting failure mode of the rolling element inner and outer ring raceway. According to current working conditions and empirical data of the wind turbine transmission system during operation, the reliability of the input and output shaft, planetary carrier, planetary shaft and spline connection in the system are generally very high, so calculation of the reliability of the transmission system mainly considers the influence of the gear and bearing. It is defined that as long as one of the three planetary gears in the row star drive fails, the row star drive cannot work normally. In this way, the failure mode of the whole wind power gearbox system is divided into sixteen failure unit events. Therefore, the reliability block diagram of the wind power gearbox is shown in Fig. 2.

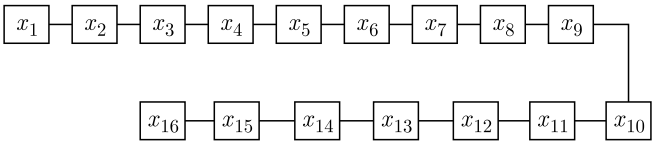

Fig. 2. Reliability block diagram of the gearbox

Here, 1-16 is internal gear 1, solar gear 2, planetary gears 3-5, helical gears 6-7, helical gears 8-9 and rolling bearings 10-16.

3. Stress and strength distribution of components

3.1. Fatigue reliability analysis of the gear root

For an involute spur gear, the tangential load is the circumferential force, which is a quotient of torque and radius. For the multiple gears split, according to equal division, the tangential loads of wind power gears in Fig. 1 are calculated according to GB/T3480-1997, as shown in Table 1.

Table 1. Tangential loads of gears in the wind power gearbox

Tangential load	s and p	p and r	1 and 2	3 and 4
F_τ	$2T_s/(n_w d_s)$	$\approx 2T_s/(n_w d_s)$	$2T_4/d_4$	T_6/d_6

Here, n_w is the number of planetary gears, d_i is the dividing circle diameter of gears, and T_i is the output torque of gears.

The diameter of the gear pitch circle and the tooth load distribution coefficient can be calculated, the service coefficient and dynamic load coefficient can borrow dynamic characteristics of the existing wind power gearbox, and the other coefficients can be found in the mechanical design manual. Each correction coefficient under known conditions is calculated according to the GB/T3480-1997. According to the ISO 6336 standard, the calculation formulas of contact fatigue stress and strength of the gear are

$$\begin{aligned} S_H &= \frac{K_A K_V K_{F\beta} K_{F\alpha} F_\tau}{b m_n} Y_{F\alpha} Y_{s\alpha} Y_\varepsilon Y_\beta = w F_\tau K_A \\ \sigma_H &= \sigma_{F\,lim} Y_N Y_S Y_{\delta rel} Y_{R\,rel} Y_X \end{aligned} \tag{3.1}$$

Here, S_H is the root bending stress, σ_H is the bending fatigue strength, w is a constant, K_A is the usage coefficient, K_V is the dynamic load factor, b is the working tooth width, m_n is the normal modulus, $K_{F\beta}$ is the load distribution coefficient along the tooth direction, $K_{F\alpha}$ is the load distribution coefficient between teeth, F_τ is the tangential load, $Y_{F\alpha}$ is the tooth top coefficient, $Y_{s\alpha}$ is the top stress correction coefficient, Y_ε is the coincidence coefficient, Y_β is the helix angle coefficient, $\sigma_{F\lim}$ is the nominal bending fatigue limit stress of the test tooth root, Y_S is the correction coefficient of the tested gear, Y_N is the life coefficient, $Y_{\delta\,rel}$ is the sensitivity coefficient of the relative root fillet, $Y_{R\,rel}$ is the surface condition coefficient of the relative root, Y_X is the size coefficient.

Setting the standard deviation of the transmission power P according to the 3σ principle, the mean and standard deviation of the tangential load on the gear can also be calculated, and then the mean and standard deviation of each stress distribution of the gears can be found. In mechanical design, it is generally believed that the above relevant random parameters all obey the normal distribution, and then the stress and strength also obey the normal distribution.

3.2. Reliability analysis of the rolling bearing

Under the action of an external random wind load, the contact stress of the rolling bearing in the wind power gearbox is alternating, and the contact fatigue failure is the main failure form of the rolling bearing. A large number of tests show that the contact fatigue life of rolling bearing obeys the Weibull distribution (Wu and Li, 2002), so the corresponding time-varying reliability evaluation model is

$$R_{bp}(t) = \exp\Big[-0.10536\Big(\frac{t}{t_{90}}\Big)^q\Big] \qquad t_{90} = \frac{16670}{n_b}\Big(\frac{C_z}{f_b P}\Big)^\varepsilon \tag{3.2}$$

Here, $q = 3/2$, $\varepsilon = 10/3$, C_z is the rated dynamic load, n_b is the working speed of the bearing, P is the equivalent momentum load, f_b is the load bearing coefficient.

According to the modern mechanical design manual, the fatigue characteristics δ_{-1} of bearing steel can be found, so the fatigue strength of the bearing is

$$\delta_{bp} = \frac{\delta_{-1}\beta_q}{K_{\delta c}} \tag{3.3}$$

Here, δ_{-1} is the fatigue strength of standard specimens, β_q is the reinforcement coefficient, $K_{\delta c} = (K_\delta/\varepsilon_\delta) + (1/\beta) - 1$ is the comprehensive correction coefficient, K_δ, ε_δ, β are selected according to the conventional design.

4. Time-varying reliability of the gearbox with failure correlation

Due to the coupled action and load correlation between the components of the wind power gearbox system, it is impossible to directly multiply each component by using the independent series theory to calculate the reliability of the gearbox system, nor to solve its reliability by using the completely related weak link theory. At the same time, the load borne by the wind power gearbox system is random and repeated for many times, and the influence of load action times on the system reliability must be considered. According to the method in (Wang *et al.*, 2010), the random variable of maximum stress S_{max} as the equivalent load of the dynamic stress stochastic process $S(t)$ in the design reference period T is calculated by sequential statistical theory. So, we can obtain the probability distribution function, the mean and the variance of S_{max}.

We assume that there is a linear correlation between different load stresses borne by each gear. Not considering strength degradation, the strength of each component is independent. For the load and strength are non-normal distributions, they are transformed into approximate normal distributions and normalized. The reliability model (Xie *et al.*, 2009) of a wind power gearbox system considering failure correlation when different structural components bear the same external load is obtained, and the reliability expression is

$$R^n = \int_0^{+\infty} f_0(S) \prod_{i=1}^{n} \left(\int_{u_i+\sigma_i S}^{+\infty} g_{min}(\delta)\, d\delta \right) dS \tag{4.1}$$

Here, $g_{min}(\delta) = PDF(\min(\delta_1, \delta_2, \delta_n))$ is the probability density function of minimum order--strength statistics, $f_0(S)$ is the standard normal probability density function, u_i, σ_i are the mean and standard deviation of stress distribution.

So, the time-varying reliability and failure rate of the wind power gearbox system in Fig. 1 are

$$\begin{aligned} R(m) &= \left[\int_0^{+\infty} m[F_0(S)]^{m-1} f_0(s) \prod_{i=1}^{16} \left(\int_{\mu_i+\sigma_i S}^{+\infty} g_{min}(\delta)\, d\delta \right) dS \right] \\ &= \left(\int_0^{+\infty} m[F_0(S)]^{m-1} f_0(S)[1 - G_{min}(\mu_1 + \sigma_1 S, \mu_2 + \sigma_2 S, \dots, \mu_{16} + \sigma_{16} S)] \right) \end{aligned} \tag{4.2}$$

and

$$\begin{aligned} G_{min}(\delta_1, \delta_2, \dots, \delta_n) &= 1 - \prod_{i=1}^{n} [1 - G_i(\delta_i)] \\ \lambda(m) &= \frac{-R'(m)}{R(m)} \end{aligned} \tag{4.3}$$

Here, $R(m)$ is the reliability function, $\lambda(m)$ is the failure rate function, m is the number of peak occurrences in T, in this paper $m = t$, $G_{min}(\delta)$ is the probability distribution function of minimum order-statistical strength, $F_0(S)$ is the standard normal probability distribution function of stress.

For comparison, assuming that all loads and strengths are independent of each other, the time-varying reliability of the wind power gearbox under random wind loads is as follows

$$R'(t) = \prod_{i=1}^{9} \left(\int_0^{+\infty} f_{s_{maxi}}(s) \int_s^{+\infty} g_i(\sigma)\, d\delta \right) \prod_{i=1}^{7} R_{bpi}(t) \tag{4.4}$$

5. Example analysis

For convenience, the seven rolling bearings are selected to be of the same type 23076CC/W33. Specific physical and geometric design parameters of a MW type of the wind power gearbox are given in reference (An *et al.*, 2013). According to the method in this paper and the mechanical design manual, the calculation results of dynamic parameters of the wind power gearbox are brought into Table 2.

Considering the repeated action of loads and the failure correlation of gears, the results of Table 2 are brought into Eqs. (4.2) and $(4.3)_2$ according to the method presented in this paper. Through numerical integration, the first-second moment method and the perturbation method (Wang *et al.*, 2013; Liu *et al.*, 2016; Hu *et al.*, 2014), the change laws of the time-varying reliability and failure rate of the wind power gearbox during the working life are calculated, as shown in Fig. 3.

Table 2. Distribution parameters of the wind power gearbox

Name	Sun gear	Planet gear	Inner gear	Helical gear 1	Helical gear 2	Helical gear 3	Helical gear 4
Mean stress u_{s_H}	450.7	505.7	604.3	644.3	704.4	506.9	501.2
Standard deviation stress σ_{s_H}	2.3	2.6	3.2	3.3	3.6	1.3	1.3
Fatigue strength u_{σ_H}	754.5	754.5	627.4	754.5	754.5	754.5	754.5
Standard deviation strength σ_{σ_H}	25.7	20.6	25.7	25.7	25.7	25.7	25.7
Rolling bearing	Contact stress				$N(454, 342)$		
	Fatigue strength				$N(875, 502)$		
	Reliability function				$R(t) \approx \exp[-10^5(t/87)^{3/2}]$		

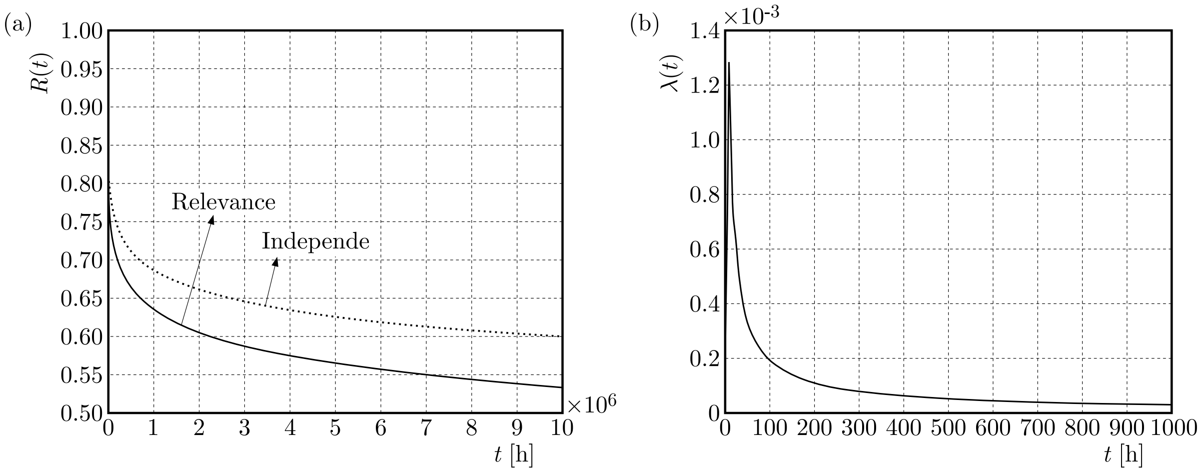

Fig. 3. (a) The time-varying reliability of system, (b) failure rate of the system

As can be seen from Fig. 3a, the reliability of the wind power gearbox gradually decreases with time, and the decreasing slope first grows and then diminishes. Comparing with the independent failure of components, the reliability of the wind power gearbox has a positive correlation with the correlation of components, that is, the better correlation, the lower reliability. With an increase of time, the impact of correlation is more obvious. As can be seen from Fig. 3b, the failure rate curve of the wind power gearbox shows characteristics of an early failure period and accidental failure period in the "bathtub" curve, that is, it gradually decreases and tends to be stable with an increase of time, so the failure is in line with life characteristics of typical mechanical products.

In order to verify the accuracy of calculation results, the Monte-Carlo method is used to determine the reliability of the gear and bearing respectively, and according to the reliability formula of a series system (Xie *et al.*, 2016, 2017), the reliability of the gearbox system is presented in Table 3.

Table 3. The result of Monte-Carlo simulation

Number of times	Reliability
One hundred thousand	0.865442
Two hundred thousands	0.865421
Five hundred thousands	0.865386
One million	0.865342
Two millions	0.865321
Five millions	0.865321

As can be seen from Table 3, with an increase of number of times, the reliability tends to be stable numerically, and the larger the number of simulation time is, the closer is the result of reliability to reality. When the number of times reaches two millions, that is, the time t is 56 h, the reliability value is close to the calculation result of Eq. $(4.3)_1$ (when the time t is 56 h, the value of the integral formula is 0.8653), so the above calculation model is effective.

6. Conclusion

Under the action of an external random wind load, gear meshing forces and rolling bearing contact forces of a wind power gearbox change with time, and its reliability index should be time--varying. Moreover, there is a statistical correlation between failure modes of various components of the system. An estimation method for time-varying reliability of the wind power gearbox with failure correlation is proposed. The main contents are: (1) Taking the output power of the wind power gearbox as the starting point, the transmission power of various components in the gearbox is calculated according to the mechanical design manual. Then through the relationship between power and load, stress distribution parameters of the failed components are found. (2) According to the stress-strength interference theory at the system level and the sequential statistical method in a random process, a reliability model of the wind power gearbox considering both failure correlation and time-varying is established. The function curves of reliability and failure rate are solved by numerical integration. (3) Comparing with the system reliability of the independent failure of components, it can be seen that the correlation between components greatly reduces reliability of the system, that is, the failure of one unit in the gearbox will accelerate the failure of another unit. (4) The reliability of the wind power gearbox working continuously for 56 hours is 86.53%, and characteristics of the failure rate conform to the life "bathtub curve" of typical mechanical products.

By making use of modern manual for mechanical design, a practical method for evaluation the reliability without complex calculation is presented in this paper. Further, we can continue to study the reliability of the wind power gearbox assuming that there is correlation between strength degradation of the components.

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